

Selection of Mobile Base Station Location Using the Speech Quality as Primary Factor

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Abstract. The selection of base station location is one of the most important decision issues for mobile operators. We propose in this paper a new multiple criteria decision-making method in order to solve the location of base station problem under fuzzy environment. In the proposed method, the ratings of each alternative and the weight of each criterion are described by linguistic variables which can be expressed by triangular fuzzy numbers. The final evaluation value of each base station (BS) location is also expressed by a triangular fuzzy number. By calculating the difference of final evaluation value between each pair of BS locations, a fuzzy preference relation matrix is constructed to represent the intensity of the preferences of one location over another. We have simulated the measurement of speech quality and the BER, and took them as decision factors; then we solved different examples of decision-making for selecting the base station location.

Keywords: Fuzzy decision making, selection of the best alternative, speech quality, bit error rate.

1 Introduction

Base station (BS) location is a common problem faced by mobile operators, in terms of coverage and better signal. In recent years, increased use of cell phones has focused attention on base stations location. Base station is viewed as a tool for gaining more customers and increasing the quality of service. In order to cover larger area, and to strengthen the signal, selecting a suitable BS location has become one of the most important decision issues for mobile operators. In the process of selection it is necessary first and foremost to identify the set of influential factors relevant to the BS location selection. Many influential factors are considered for the selection of a particular BS location, e.g. investment cost, area coverage, covered population, speech quality, etc. [1]. Multiple criteria decision-making (MCDM) methods were provided to deal with the problem of ranking and selecting the BS location under multiple criteria [2]. In order to estimate the speech quality, test measurement of the transferred signal quality must be performed in aspect of BER and RxQual parameters. The parameters have to be measured on specific locations in the range of

the each BS that is subject of evaluation using specialized equipment (mobile BTS) or radio network planning software with detailed digital maps of the area.

In general, the selection of a best BS location from among two or more alternative locations on the basis of two or more factors is a multi-criteria decision-making problem. Under many situations, the values for qualitative criteria are often imprecisely defined for the decision-makers. Besides, the desired value and importance weight of criteria are usually described in linguistic terms, e.g. "very low", "medium", "high", "fair", etc. It is not easy to precisely quantify the rating of each alternative location and the precision-based methods as stated above are not adequate to deal with the plant location selection problem. This fuzziness in the BS location selection process motivated us to develop a fuzzy decision-making method.

By using the pair-wise preference relations, we present a new fuzzy decision-making method to deal with BS location selection problem in this paper [6]. The decision-making criteria are divided into quantitative and qualitative criteria in our method. The importance weights of decision criteria and the ratings of qualitative criteria are assessed in linguistic variables which are described by triangular fuzzy numbers. In the proposed method, we aggregate the ratings (fuzzy and crisp) and fuzzy weights to calculate the final fuzzy evaluation values of all candidate locations. A preference relation is defined to indicate the over degree of preference of each pair of BS locations by comparing the difference between their final fuzzy evaluation values for all possibly occurring combinations. According to the preference relations, we construct a fuzzy preference relation matrix and use a stepwise ranking procedure to determine the ranking order of a large number of plant locations.

The organization of this paper is as follows. First, we introduce the basic definitions and notations of fuzzy numbers and linguistic variables. Next, we define a fuzzy preference relation to derive the fuzzy preference relation matrix, and propose a stepwise ranking procedure to determine the ranking order of all BS locations. Then we give an overview of speech quality measurement, and an example is solved in Matlab to illustrate the working of the proposed method. Finally, we give some conclusions at the end of this paper.

2 Fuzzy Decision Making

In this section, a systematic approach to the BS location selection problem by using the concepts of fuzzy set theory and multiple-criteria decision analysis is proposed. This method is very suitable for decision-making under fuzzy environment. Knowing the fuzziness of the BS location selection problem, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic (fuzzy) variables in this paper.

The linguistic variables can be expressed as triangular fuzzy numbers – given in Tables 1 and 2. We suggested that the decision maker easily uses the linguistic variables (shown in Table 1 and 2) to evaluate the importance of the criteria and the ratings of alternatives with respect to various subjective criteria.

Let $A_1, ..., A_m$ be possible alternatives (number of feasible BS locations) and $C_1, ..., C_n$ be criteria with which alternative performances are measured. As stated above, a



fuzzy multi-criteria decision-making method for the selection of BS location problem can be concisely expressed in matrix format as:

$$\underline{D} = \begin{bmatrix} \underline{x}_{11} \, \underline{x}_{12} & \cdots \underline{x}_{1n} \\ \underline{x}_{21} \, \underline{x}_{22} & \cdots \underline{x}_{2n} \\ \vdots & \vdots & \cdots \\ \underline{x}_{m1} \, \underline{x}_{m2} & \cdots & \underline{x}_{mn} \end{bmatrix} \quad \underline{W} = [\underline{w}_{1}, \, \underline{w}_{2}, \, \dots, \, \underline{w}_{n}]$$

where \underline{x}_{ij} , $\forall i,j$ is the fuzzy rating of alternative A_i (i=1, 2, ..., m) with respect to criterion C_j and \underline{w}_j (j=1, 2, ..., n) is the weight of criterion C_j . These fuzzy ratings and the weights of each criterion are linguistic variables which can be described by triangular fuzzy numbers, $\underline{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\underline{w}_j = (w_{il}, w_{i2}, w_{j3})$.

Therefore, we can obtain the normalized fuzzy decision matrix denoted by $\underline{\mathbf{R}}$ as:

$$\underline{R} = [\underline{r}_{ij}]_{\text{mxn}}$$

$$\underline{r}_{ij} = (a_{ij}/c_{j}^{*}, b_{ij}/c_{j}^{*}, c_{ij}/c_{j}^{*}), j \in B$$

$$\underline{r}_{ij} = (a_{j}/c_{ij}, a_{j}/b_{ij}, a_{j}/a_{ij}), j \in C$$

$$c_{j} *= max_{i} c_{ij}, if j \in B, a_{j} = min_{i} a_{ij}, if j \in C$$
(1)

where B and C are the set of benefit criteria and cost criteria, respectively.

The normalization method mentioned above is to preserve the property that the ranges of normalized fuzzy numbers belong to [0, 1].

Considering the different importance of each criterion, we calculate the final fuzzy evaluation value of each alternative as:

$$\underline{P}_{i} = \sum_{i=1}^{n} \underline{r}_{ij}(\cdot)\underline{w}_{j}, i=1, 2, ..., m$$
(2)

where \underline{P}_i is the final fuzzy evaluation value of alternative A_i . After the calculation of the final fuzzy evaluation value of each alternative, the pair wise comparison of the preference relationship between the alternatives A_i and A_j can be established as stated in the following section.

To define a preference relation of alternative A_i over alternative A_j we do not directly compare the membership function of \underline{P}_i and \underline{P}_j . Instead, we use the membership function of $\underline{P}_i(-)\underline{P}_j$ to indicate the preferability of alternative A_i over alternative A_j and then compare $\underline{P}_i(-)\underline{P}_j$ with zero.

Here, the final fuzzy evaluation values \underline{P}_i and \underline{P}_j are triangular fuzzy numbers. The difference between \underline{P}_i and \underline{P}_j is also a triangular fuzzy number and can be calculated as:

$$\underline{Z}_{ij} = \underline{P}_{i}(-)\underline{P}_{j} \tag{3}$$

$$\underline{Z}^{\alpha}_{ij} = \left[z^{\alpha}_{ijl}, z^{\alpha}_{iju} \right] \tag{4}$$

$$\underline{P_{i}}^{\alpha} = [p_{ib}^{\alpha} p_{iu}^{\alpha}], \underline{P_{j}}^{\alpha} = [p_{jb}^{\alpha} p_{ju}^{\alpha}], z_{ijl}^{\alpha} = p_{il}^{\alpha} p_{ju}^{\alpha}, z_{ijl}^{\alpha} = p_{ju}^{\alpha}, z_{iju}^{\alpha} = p_{iu}^{\alpha} p_{ju}^{\alpha}$$

If $z^{\alpha}_{ijl} > 0$ for $\alpha \in [0, 1]$, then alternative A_i is absolutely preferred to A_j . If $z^{\alpha}_{ijl} < 0$ for $\alpha \in [0, 1]$, then alternative A_i is not absolutely preferred to A_j . If $z^{\alpha}_{ijl} < 0$ and $z^{\alpha}_{ijl} > 0$ for some α values, we define e_{ij} as a fuzzy preference relation between alternatives A_i and A_j to represent the degree of preference of alternative A_i over alternative A_j . The e_{ij} is defined as:

$$e_{ij} = S_1/S, S > 0, S_I = \int_{x>0} \mu_{\underline{Z}ij}(x) dx, S_2 = \int_{x<0} \mu_{\underline{Z}ij}(x) dx, S = S_I + S_2$$
 (5)

The value of e_{ij} is the degree of preference of alternative A_i over alternative A_j and $\mu_{Zij}(x)$ is the membership function of $\underline{P}_j(-)\underline{P}_j$.

Intuitively, S_I indicates the portion where alternative A_i is preferred to alternative A_j in the most favorable situation. The e_{ij} indicates the over degree of preference of alternative A_i over alternative A_j . An illustration of calculating e_{ij} is shown on Fig.1. Therefore, $e_{ij} > 0.5$ indicates the alternative A_i is preferred to alternative A_j . If $e_{ij} = 0.5$ then there is no difference between alternatives A_i and A_j . If $e_{ij} < 0.5$ then alternative A_j is preferred to alternative A_i .

Table 1. Linguistic variables for the importance weight of criterions

Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very high (VH)	(0.9, 1.0, 1.0)

Table 2. Linguistic variables for the ratings

Very poor (VP)	(0,0,1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

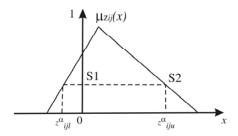




Fig. 1. An illustration of calculating e_{ii}

Using the fuzzy preference relation, we can construct a fuzzy preference relation matrix as:

$$E = [e_{ii}]_{m \times m} \tag{6}$$

The fuzzy preference relation matrix represents the degree of preference of each pair alternatives. According to the fuzzy preference relation matrix E, the fuzzy strict preference relation matrix can be defined as:

$$\mathbf{E}^{\mathbf{S}} = [e^{\mathbf{S}}_{ij}]_{\mathbf{m} \times \mathbf{m}} \tag{7}$$

$$\begin{cases} e_{ij} - e_{ji}, when \ e_{ij} \ge e_{ji} \\ 0, \quad otherwise \end{cases}$$
(8)

The value of e^{S}_{ij} is a degree of strict dominance of alternative A_i over alternative A_j . Then, the non-dominated degree of each alternative A_i (i=1, 2, ..., m), can be determined by using the fuzzy strict preference relation matrix as:

$$\mu^{ND}(A_i) = \min_{i \in \Omega} \{ 1 - e^{S_{ij}} \} = 1 - \max_{i \in \Omega} e^{S_{ii}}$$
 (9)

where $\mu^{ND}(A_i)$ is the non-dominated degree of each alternative A_i and Ω is a set of alternatives. Therefore, we can use the $\mu^{ND}(A_i)$ values to rank a set of alternatives. The ranking procedure is described as follows:

- (i) Set K=0 and $\Omega = \{A_1, ..., A_m\}$.
- (ii) Select the alternatives which have the highest non-dominated degree, say A_h , $\mu^{ND}(A_h) = \max_i \{\mu^{ND}(A_i)\}$. Set the ranking for A_h as $r(A_h) = K+1$.
- (iii) Delete the alternatives A_h from Ω , i.e. $\Omega = \Omega M_h$. The corresponding row and column of A_h are deleted from the fuzzy strict preference relation matrix [7].
- (iv) Recalculate the non-dominated degree for each alternative A_i , $A_i \in \Omega$ If $\Omega = \emptyset$, then stop. Otherwise, set K = K + 1, and return to step (ii).

3 Speech Quality as a Decision Factor

In order to estimate the speech quality regarding BER (bit error rate) and RxQual parameters, on-field measurement using specialized equipment (mobile or moveable BTS) could be performed, or simulation of the radio signal propagation with appropriate model for urban / rural areas as we have done.

In GSM, bit error rate (BER) measurements are used to decide whether transmitter power should be changed and in deciding whether a call should be attached to another base station. A single BER measurement in GSM is reported as one of the eight quality levels (RXQUAL 0...7) which is estimated by backward coding of the

decoded bit sequence and comparing it to the received bit sequence. This is a measure of the raw bit error rate, and does not take into consideration channel coding and the used speech codec. In this simulation - BER is estimated using AMR-NB codec with appropriate channel coding [3].

The received speech is compared with the test sequence transmitted between the BS and the receiver (downlink) in a similar way as the human speech perception and the quality is graded, (the listeners should do it in traditional subjective tests, like MOS). Example of one of the most popular used algorithms for intrusive tests in packet switched and mobile networks is PESQ, defined in P.862 ITU-T [4]. PESQ is capable to predict subjective quality expressed by MOS values with good correlation in a very wide range of conditions, which may include coding distortions, errors, noise, filtering, delay and variable delay.

To justify the use of these two criteria several tests were performed with simulation testbed of the GSM transmission path. On Fig. 2 is presented BER plot of the simulated transmission path and measured MOS score. It could be noticed that these two parameters are loosely correlated, there is no linear relationship between them, and sometime good BER could produce unrecognizable decoded speech and vice versa. The location of the errors within speech frame has influence on the perceived speech quality as well.

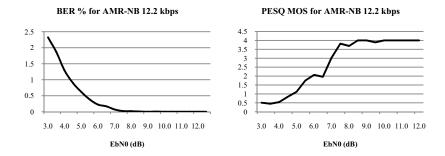


Fig. 2. BER and PESQ plot of the simulated GSM transmission path

AMR-NB (Adaptive Multi-Rate – Narrow Band, ACELP) is used for speech codec in these tests. It is standardized by ETSI for GSM applications, and it is chosen as mandatory for 3GPP networks [5]. AMR-NB is a speech codec with 8 different narrowband modes of operation and data rates between of 4.75 and 12.2 Kb/s. In the simulations, the rate of 12.2 Kb/s which is compatible with GSM-EFR codec (3GPP TS 26.071) is used. This speech codec is mainly used for toll quality speech compression in the 2nd and 3rd generation mobile telephony applications.

The system that is used for simulation is designed and coded in Matlab, it allows simulation of reference sequence transmission over the communication link with packet or frame loss events at the receiver side. Then comparison between the reference and received speech sequence is done and MOS score is evaluated.

Packet or frame loss is a major source of speech impairment and GSM applications. The impact of packet loss in perceived speech quality depends on



several factors, including loss pattern, codec type, and packet loss size. It may also depend on the location of loss within the speech. Even more - as most real communication channels exhibit burstiness of packet loss, occurrence of burst of lost packets has significant impact over speech perception.

The resulting data stream is then protected by an error-control coding scheme according technical specification [3]. On the radio transmission path, various sources of errors can disturb the transmitted data and at the receiver, the channel decoder attempts to recover from these errors and delivers a "cleaned up" version of the received data. Finally, speech is reconstructed in the speech decompression block.

In Table 3, BER and MOS values are shown, produced by simulation of transmission path (with simplified Rayleigh fading channel) with 3 different stations on 4 measurement points for given different EbN0 - energy per bit to noise power spectral density ratio.

	В	S1	BS	52	BS	53
Eb/N0	RxQ (BER)	PESQ	RxQ (BER)	PESQ	RxQ (BER)	PESQ
6 dB	1,69%	2,10	2,52%	1,56	2,90%	1,36
7 dB	1,07%	2,08	0,96%	1,37	1,30%	2,14
8 dB	0,37%	2,33	0,48%	1,94	0,71%	2,40
9 dB	0,09%	3,02	0,22%	1,36	0,44%	2,49

Table 3. Simulated measurements on 4 points per BS location (makro-cells)

4 Simulation Results

We have developed application in Matlab that enables selection of a location for establishing a new base station. A graphical output will be obtained in order to enable easy and effective application of our work for end-users. We will illustrate a problem with three decision-makers D_1 , D_2 and D_3 , three alternative locations, and five decision criteria. After preliminary screening, three candidate-sites A_1 , A_2 and A_3 remain for further evaluation. The company considers the following five criteria to select the most suitable location:

- (1) investment cost (C_1) ,
- (2) area coverage (C_2) ,
- (3) population covered (C_3) ,
- (4) BER-RxQual (C₄),
- (5) Speech quality PESQ (C_5),

The benefit and cost criteria set are $B = \{C_2, C_3, C_4, C_5\}$ and $C = \{C_I\}$, respectively. The hierarchical structure of this decision problem is shown in Fig. 3.

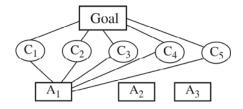


Fig. 3. The hierarchical decision structure.

The proposed method is currently applied to solve this problem. The computational procedure is summarized as follows:

- Step 1: The decision-makers use the linguistic weighting variables (from Table 1) to assess the importance of the criteria and present it in Table 4. The fuzzy weight of each criterion calculated is given in Table 5.
- Step 2: The decision-makers use the linguistic rating variables (shown in Table 2) to evaluate the rating of alternatives with respect to each criterion and reset in Table 6
- Step 3: According to Table 5, the fuzzy decision matrix is constructed as shown in Table 7.
 - Step 4: Construct the fuzzy normalized decision matrix as shown in Table 8.
 - Step 5: The final fuzzy evaluation values of three alternatives are calculated as:

$$\underline{P}_{I}$$
=(3.03, 4.17, 4.74)
 \underline{P}_{2} =(2.60, 3.83, 4.50)
 \underline{P}_{3} =(2.57, 3.67, 4.37)

Step 6: The difference between two final fuzzy evaluation values are calculated as:

$$\underline{P}_1(-)\underline{P}_2 = (-1.47, 0.34, 2.14)$$

 $\underline{P}_1(-)\underline{P}_3 = (-1.34, 0.50, 2.17)$
 $\underline{P}_2(-)\underline{P}_3 = (-1.77, 0.16, 1.93)$

Step 7: Construct the fuzzy preference relation matrix:

$$\mathbf{E} = \begin{bmatrix} 0.50 & 0.66 & 0.72 \\ 0.33 & 0.50 & 0.56 \\ 0.28 & 0.44 & 0.50 \end{bmatrix}$$

Step 8: Construct the fuzzy strict preference relation matrix as:

$$\boldsymbol{E^{S}} = \begin{bmatrix} 0 & 0.33 & 0.44 \\ 0 & 0 & 0.13 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 9: Compute the non-dominated degree of each alternative A_i (i=1, 2, 3) as: $\mu^{ND}(A_1) = 1.00$; $\mu^{ND}(A_2) = 0.67$; $\mu^{ND}(A_3) = 0.56$;

Table 4. The importance weight of the criteria

	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3	
C_1	Н	VH	VH	
C ₂	MH	Н	MH	
C ₃	Н	Н	Н	
C ₄	Н	Н	Н	
C ₅	VH	VH	VH	



Table 5. The fuzzy weights of the criteria

	C_1	C_2	C_3	C ₄	C ₅
Weight	(0.8, 0.9, 1.0)	(0.57, 0.77, 0.93)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.9,1.0,1.0)

Table 6. The ratings of the three candidates by decision-makers under all criteria

Criteria	Candid.	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3
C_1	A_1, A_2, A_3	3, 6, 4 [mil]	5, 8, 6 [mil]	4, 7, 5 [mil]
C_2	A_1, A_2, A_3	G, VG, MG	VG, VG, G	F, VG, VG
C ₃	A_1, A_2, A_3	F, G, G	G, G, MG	G, G, VG
C ₄	A_1, A_2, A_3	VG, G, F	VG, G, F	VG, G, F
C ₅	A_1, A_2, A_3	VG, G, VG	VG, G, VG	VG, G, VG

Table 7. The fuzzy decision matrix

	\mathbf{C}_{1}	C ₂	\mathbb{C}_3	C_4	C ₅
\mathbf{A}_{1}	4 m.	(6.3, 9, 7)	(5.7, 7, 7)	(9, 7, 3)	(9, 7, 9)
\mathbf{A}_{2}	7 m.	(8, 10, 8.7)	(7.7, 9, 8.7)	(10, 9, 5)	(10,9,10)
A ₃	5 m.	(9, 10,9.7)	(9, 10,9.7)	(10, 10, 7)	(10,10,10)

Table 8. The fuzzy normalized decision matrix

	C_1	C_2	C_3	C_4	C ₅
\mathbf{A}_{1}	1	(0.6, .9, .7)	(0.6,0.7,0.7)	(0.9,0.7,0.3)	(0.9,0.7,0.9)
A_2	0.6	(0.8,1,.9)	(0.8,0.9,0.9)	(1,0.9,0.5)	(1, 0.9, 1)
A ₃	0.8	(0.9, 1, 1)	(0.9, 1, 1)	(1, 1, 0.7)	(1, 1, 1)

Step 10: The alternative A_1 has the highest non-dominated degree and set $r(A_1)=1$.

Step 11: Delete the alternative A_1 from the fuzzy strict preference relation matrix.

Step 12: After deleting the alternative A_I , the new fuzzy strict preference relation matrix is:

$$\mathbf{E}^{S} = \begin{matrix} A_{2} & A_{3} \\ A_{2} & 0 & 0.13 \\ A_{3} & 0 & 0 \end{matrix} \right]$$

The non-dominated degree of alternatives A_2 and A_3 are 1.0 and 0.87 respectively. Therefore, $r(A_2)=2$ and $r(A_3)=3$. The ranking order of the three alternatives is $\{A_I\}>\{A_2\}>\{A_3\}$. Therefore, the site A_I is the best location to establish a new base station. We can see that the proposed method not only allows decision-makers to determine the ranking order of alternative alternatives but also can indicate the degree of preference of each pair of alternatives. Therefore, it is more suitable and effective in dealing with subjective judgments in an imprecise environment.

5 Conclusion

In this paper we have proposed a new fuzzy multiple criteria group decision-making method for solving the problem of base station (BS) location. In BS location selection, very often, the assessment of alternatives with respect to criteria and the importance weight of criteria are given in linguistic variables. We have presented a stepwise and objective method to determine the ranking order of fuzzy numbers.

In this paper – a systematic and objective method is proposed to deal with BS location selection problem. The proposed method can help the decision-maker to make a suitable decision under fuzzy environment. In order to estimate the speech quality, on-field measurement of the transferred signal quality must be performed in aspect of BER and MOS parameters. The parameters have to be measured on specific locations in the range of the each BS that is subject of evaluation.

We have realized simulation in Matlab, and solved different examples of decisionmaking tasks for selecting the base station location. We've illustrated a problem with three decision-makers, three alternative locations, and five decision criteria. A graphical output is obtained in order to enable easy and effective application of our work for end-users.

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