A New Networked Filter Method for Robust Stability Analysis of Software Defined Networks

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Abstract. In this paper, we propose a new filtering method for robust stability analysis of software defined networks (SDNs) with nonsmooth features by using the formal specification of an abstract network architecture of SDNs in which the convergence of the delay time equals to the fairness and the convergence problem of delay time of SDNs is transformed into a robust stability problem. Our theoretical result for robust stability analysis of SDNs with respect to the convergence shows that this mathematical filtering method is promising for the analysis of SDNs.

Keywords: Software defined network, filter theory, nonsmooth analysis.

1 Introduction

The nonsmooth characteristics of software defined networks (SDNs) [1], can be formalized by the networked filter based on the network abstraction of the robust SDNs. The fairness of the cloud service in SDNs is reflected in the convergence of delay time of channels among different data centers and the converged responses represented by delay time indicate the proportional fairness of services.

In order to characterize the nonsmoothness of SDNs, we propose a signal processing method for the convergence problem of delay time of SDNs, which is equivalent to the robust stability problem.

2 Mathematical Modeling

In our study, the convergence of the delay time of the channels that corresponds to the fairness of the response time for users who use the cloud services of the SDNs, is transformed into the dual form of the fairness problem formulated as the robust stability of observed signals under the condition that the delay time of the channels is converged within certain threshold. So, the filtering process of the state $X(t)$ regarded as a Wiener filtering process, can be approximated by the equivalent problem of the filtering of $X(t)$ given in the generic form of a networked filter we designed, where $t \geq 0$. This filtering process under the nonsmoothness of $X(t)$ caused by the state transition among different steady states (SSs) is regarded as a nonsmooth analysis problem in mathematics.

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We define the notations as follows: k is the index of the state that refers to the SS observed at the current step of the filtering process ($k \geq 1$); k-1 is the index of the state that refers to the SS observed at the previous step of the filtering process; k+1 refers to the index of the state that refers to the SS observed at the next step of the filtering process; $q_{k-1,k}$ is the probability of the state transition from the state $X(k-1|t)$ to the state $X(k|t)$; $q_{k,k}$ is the probability for maintaining the state $X(k|t)$; $q_{k,k+1}$ is the probability of the state transition from the state $X(k|t)$ to the state $X(k+1|t)$. We assume that $q_{k-1,k} = q_{k,k+1} = q$ considering the condition that no bias exists among different SSs, then

$$q_{k,k} = e^{2h_0 t}, \tag{1}$$

where h_0 is a constant. The estimation of q is made by using the following:

$$\int_k^{k+1} P(X(k+1|t), X(k|t)|\sigma) \ln\left(\frac{P(X(k+1|t))}{P(X(k|t))}\right) d\sigma \tag{2}$$

for

$$\Delta q_{k,k} = C(q - q_{k,k}), \tag{3}$$

where $P(X(k|t)$ and $P(X(k+1|t))$ are probabilistic functions that are defined as the Gaussian distribution with zero mean and unit variance; $P(X(k+1|t), X(k|t)|\sigma)$ is a probability of a Poisson process with the variable of $\max(|X(k+1|t) - X(k|t)|, \lambda)$ and $\lambda = 1$ under the condition that $X(k+1|t) - X(k|t)$ approaches to a supermartingale. C is a constant; the variable $\sigma \in S$ is defined as the random variable of a Wiener process that is normally denoted as W in the Ito integral and assigned with the value of the marginal distribution dependent on k. S is the area of the multiple steady states in the probabilistic space under the condition of the nonsmoothness exists among different SSs ($X(t)$ is not differential). The SS will be obtained as $t \to \infty$ and $q \to 0$, which indicates the convergence of the filtering process, that is, the equivalent process of sustaining robust stability.

3 Conclusion

This paper proposed a new signal processing method to solve the nonsmooth analysis problem of SDNs. The proposed method, which is different from the exiting numerical calculation for performance evaluation in communication engineering, is a kind of non-parametric modeling. With the proposed filtering method that uses the encapsulation of the linear function and corresponding stochastic constraints, it becomes feasible to model the network dynamics of SDNs by using the observed signals. The convergence of the delay time, which is studied in mathematics by using its dual form of robust stability, shows that the mathematical filtering method is valuable in analyzing the robust stability of SDNs.

References

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