Thermal transition in nonlinear lattices

Mihaela Carina RAPORTARU
Horia Hulubei National Institute for Physics and Nuclear Engineering
Reactorului, 30, Magurele, Ilfov, Romania

The dynamics of mesoscopic quantum systems represents a very active area of scientific research with distinct contributions from quantum physics, applied mathematics and high performance computing. Part of the appeal of these systems comes from their experimental maneuverability which allows for detailed investigations which are fully supported by numerical computations and theoretical modeling [1].

In this contribution we study a general class of stochastic ordinary differential equations which describes a large set of physical systems and show that the stability of the aforementioned physical systems depends strongly on the strength of the stochastic term [2]. In their most general form the ϕ^4 lattice equations can be cast as

$$m\ddot{x}_n = -U'(x_n) + k(x_{n+1} + x_{n-1} - 2x_n) - \alpha\dot{x}_n + \xi_n$$

where $\langle \xi_n(t)\xi_n(t')\rangle = 2D\delta_{mn}\delta(t-t')$ and $U(x_n) = a\,x_n^2/2 + b\,x_n^4/4$.

One particular application of these equation concerns the nonlinear dynamics of Bose-Einstein condensates loaded in to optical lattices, with x_n representing the number of atoms in each well of the lattice [3]. In the above equation α and ξ_n measure the damping in each well of the lattice and the level of noise due to thermal fluctuations, respectively. The solutions which are physically relevant are: that with all x_n constant (period-1 solution), that with an x_n series of the type $x_1, x_2, x_1, x_2, x_1, x_2$ ... (period-2 solutions), and $x_1, x_2, x_3, x_1, x_2, x_3 x_1, x_2, x_3$ ... (period-3 solutions). All these solutions are dynamically stable as far as deterministic stability is concerned so we will study the stability of these solutions with respect to thermal fluctuations. To this end, we will integrate numerically the previous set of equations using the Euler-Maruyama method which provides us with the time evolution of the number of particles in each well.

The main conclusion of our numerical investigation is that all previous higher-period states are unstable with respect to stochastic perturbations and decay to the fundamental period-1 solution. We illustrate this result with two typical dynamics, one for a period-2 state and one for a period-3 state, which show very clearly the decay of these states to the fundamental period-1 state. For the transition from a period-2 state to the fundamental period-1 state we have used $a = -1$, $b = 1$, $\alpha = 0.05$, and $D = 0.00125$, while for the transition from a period-3 state to the fundamental period-1 state we have used $a = -1$, $b = 1$, $\alpha = 0.01$, and $D = 0.002$.

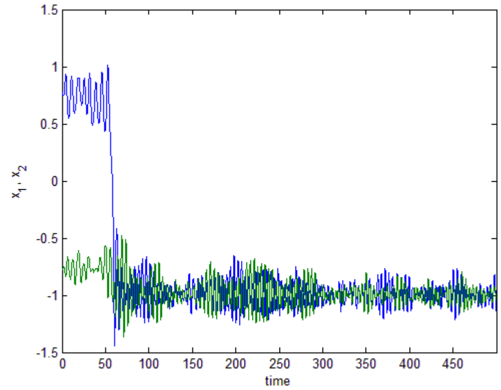

Figure 1. Transition from a period-2 state to the fundamental period-1 state. In blue x_1, in green x_2.

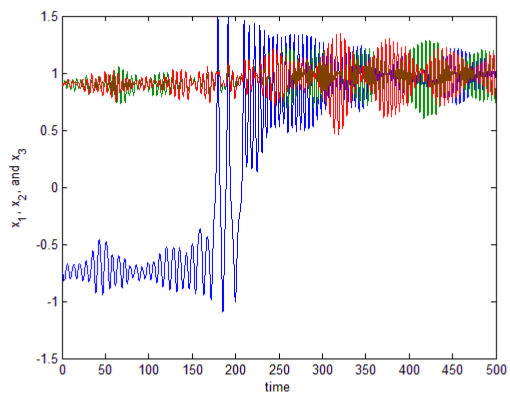

Figure 2. Transition from a period-3 state to the fundamental period-1 state. In blue x_1, in green x_2, in red x_3.

The main conclusion of our numerical investigations on the properties of the ϕ^4 lattice equations is that spatially periodic stationary solutions are unstable with respect stochastic perturbations. All of the high period states we have investigated showed a clear decay to the period-1 state in some region of the parameters of the equations, thereby indicating that this is the generic feature of the high period solutions. Our findings are relevant for Bose-Einstein condensates loaded in optical lattices as we will show elsewhere in detail.

References:

[1] M. A. Porter, N. J. Zabusky, B. Hu, and D. K. Campbell, American Scientist **97**, 214 (2009)

[2] M.A. Porter, R. Carretero-González, P.G. Kevrekidis, and B. A. Malomed, Chaos **15**, 015115 (2005)

[3] C.J. Petrick and H. Smith, *Bose-Einstein condensation in dilute gases*, Cambridge University Press (2008)